PHYSICAL REVIEW E VOLUME 57, NUMBER 4 APRIL 1998

Transport of particles caused by correlation between additive and multiplicative noise

J. H. Li[1,2,3] and Z. Q. Huang[2]
[1]Department of Physics, Nanjing University, Nanjing 210008, China
[2]Institute of Low Energy Nuclear Physics, Beijing Normal University, Beijing 100875, China
[3]National Laboratory of Solid State Microstructures, Nanjing University, Nanjing 210008, China

(Received 7 November 1997)

The transport of a spatially periodic system driven by additive and multiplicative Gaussian white noises (between which there is a correlation) is investigated in the presence of spatial symmetry. The probability current shows that the correlation between additive and multiplicative noise is an ingredient for the flux of particles. Using the formulas obtained by us we investigate the transport in the superconducting junction and the transport of motor proteins in the case of environmental perturbation. For the motor proteins we find that the multiplicative noise can also induce the flux of the moleculae, even if the correlation between additive and multiplicative noise is zero. The results of superconducting junction and motor proteins provide a theoretical foundation for further study. [S1063-651X(98)07104-9]

PACS number(s): 05.40.+j, 74.40.+k, 82.20.Mj, 87.15.Rn

I. INTRODUCTION

Recently a considerable amount of analysis has been devoted to investigating transport of particles in spatially periodic stochastic systems [1]. The hopeful observation is that nonequilibrium fluctuations can lead to transport in spatially periodic but asymmetric systems [2–8]. It has also been shown that mean-zero noise of a more complicated asymmetric type can lead to similar phenomena even in the absence of a spatial asymmetry [9–13]. It is clear that the spatial asymmetry and the noise asymmetry are ingredients for transport. In Refs. [2,13,14], it is reported that the time correlation of noise is also an ingredient for transport.

Now preliminary attempts have been made to apply these phenomena to the operation of biomolecular motors [6,15], as applications of new molecular separation techniques [16], to condensed matter type systems [17,18], and to understanding the kinetics of single ion channels [19]. Potential ratchets, fluctuating potential ratchets, and fluctuating force ratchets in the asymmetric case have been investigated [2,6,20,21]. A rocked thermal ratchet driven by a zero-mean time-periodical external force was considered in Refs. [22,23]. The transport in symmetric periodic potential systems driven by Poisson white noise (shot white noise) or asymmetric dichotomous noise has been demonstrated [13,24,25].

All of the above work has been focused on the transport of particles in the spatially asymmetric case or the noise asymmetric case. In this paper we shall investigate the transport in a spatially periodic symmetric system with symmetric additive and multiplicative noise. The main point is that the correlation between additive and multiplicative noise can cause net flow even in the absence of spatial asymmetry. The setup for the problem is as follows: We first study a general system consisting of overdamped Brownian particles whose differential equation is driven by Gaussian white noise, between which there is a correlation. Then using the formulas obtained by us we try to investigate the transport in superconducting (Josephson) junction and the transport of motor proteins with environmental perturbation.

II. THE OVERDAMPED BROWNIAN PARTICLES

The equation of the overdamped Brownian particles for the system is

$$\dot{x}=f(x)+g(x)\xi(t)+\eta(t), \quad f(x)=-\frac{dU_0(x)}{dx},$$
$$g(x)=-\frac{dU_1(x)}{dx}, \tag{1}$$

where $U_0(x)$ and $U_1(x)$ are spatially periodic functions with a period L. $\xi(t)$ (the multiplicative noise) and $\eta(t)$ (the additive noise) represent the Gaussian white noises. In general, we express the influence of the internal fluctuation on the system as additive noise and the effect of the external environmental fluctuation on the system as multiplicative noise. Here we assume that the external environmental fluctuation can influence the internal fluctuation. Because of the influence of the external environmental fluctuation on the internal fluctuation, additive and multiplicative noise are not independent (there is correlation between them). The statistical properties of $\xi(t)$ and $\eta(t)$ are $\langle \xi(t)\rangle_f=\langle \eta(t)\rangle_f=0$, $\langle \eta(t)\eta(t')\rangle_f=2D_1\delta(t-t')$, $\langle \xi(t)\xi(t')\rangle_f=2D_2\delta(t-t')$, and $\langle \xi(t)\eta(t')\rangle_f=2\lambda\sqrt{D_1D_2}\,\delta(t-t')$ [$(0\leqslant\lambda\leqslant1)$, $\langle\rangle_f$ denotes averaging over the noise]. The Stratonovich interpretation of Eq. (1) yields the Fokker-Planck equation [26,27]

$$\partial_t P(x,t)=-\partial_x J(x,t), \tag{2}$$

with the probability current $J(x,t)$ given by

$$J(x,t)=A(x)P(x,t)-\partial_x B(x)P(x,t), \tag{3}$$

where $A(x)=f(x)+D_2 g'(x)[g(x)+\lambda\sqrt{D_1/D_2}]$, and $B(x)=D_1(1-\lambda^2)+D_2[g(x)+\lambda\sqrt{D_1/D_2}]^2$. The prime indicates derivative with respect to x of the function $g(x)$.

In the stationary state, for $t\rightarrow\infty$, the distribution $P(x,t)\rightarrow P(x)$ and the current $J(x,t)\rightarrow J=\text{const}$. Then we have

$$J=A(x)P(x)-\partial_x B(x)P(x). \tag{4}$$

1063-651X/98/57(4)/3917(6)/$15.00

© 1998 The American Physical Society

However, J is not arbitrary, but is determined by normalization and periodic boundary conditions $(b-a=L)$ $P(a)=P(b)$, $J(a)=J(b)=J$. For convenience, define $\phi(x)=\int_a^x[A(x')/B(x')]dx'$. Then dividing the sides of Eq. (4) by $e^{\phi(x)}$ and noting $\partial_x B(x)P(x)/e^{\phi(x)}=\partial_x B(x)P(x)/e^{\phi(x)}+B(x)P(x)/e^{\phi(x)}\partial_x\phi(x)$, we obtain

$$\frac{J}{e^{\phi(x)}}=\frac{A(x)P(x)}{e^{\phi(x)}}-\partial_x\frac{B(x)P(x)}{e^{\phi(x)}}-\frac{B(x)P(x)}{e^{\phi(x)}}\partial_x\phi(x). \tag{5}$$

Substituting $\partial_x\phi(x)=A(x)/B(x)$ into Eq. (5), we have

$$\frac{J}{e^{\phi(x)}}=-\partial_x\frac{B(x)P(x)}{e^{\phi(x)}}. \tag{6}$$

Integrating Eq. (6) from a to b, we get

$$J=\frac{B(a)P(a)\exp[-\phi(a)]-B(b)P(b)\exp[-\phi(b)]}{\displaystyle\int_a^b\exp[-\phi(x)]dx}. \tag{7}$$

It is clear that $P(a)=P(b)$, $B(a)=B(b)$, and $\phi(a)=0$, so Eq. (7) becomes

$$J=\frac{B(a)P(a)\{1-\exp[-\phi(b)]\}}{\displaystyle\int_a^b\exp[-\phi(x)]dx}=N\{1-\exp[-\phi(b)]\}, \tag{8}$$

where $N=P(a)B(a)/\int_a^b dx\,\exp[-\phi(x)]$, which is the normalization constant for the stationary probability distribution [cf. Eq. (11)].

From Eq. (8) we find that the solution for the nonzero current J has a definite sign. This is because of the symmetry breaking, which is induced by the correlation between the additive and multiplicative noises. It is necessary to give insight into the origin of this. Consider a solution $x(t)$ (provided that it corresponds to the current J) of Eq. (1) for a given realization of the noises. Then $-x(t)$ is also a solution of Eq. (1), with t replaced by $-t$. If $\eta(t)$ and $\xi(t)$ were uncorrelated, the solution $-x(t)$ would have the same probability as $x(t)$; the corresponding current is $-J$ (as a matter of fact, now the currents $\pm J$ are zero). But when there is a correlation, the probability does not have this symmetry; the current is also so. In a sense then the phenomenon reported here is due to symmetry breaking, which is induced by the correlation between additive and multiplicative noise. In addition, from Eq. (8) we can still find that when $\phi(b)>0$, J is positive; while when $\phi(b)<0$, J is negative. Thus the condition under which J changes sign is that the value of $\phi(t)$ can vary from positive to negative (or from negative to positive).

It needs to be explained that the zero-current general solution of Eq. (4) is not admissible to the system. Indeed, $J=0$ and its corresponding solution of Eq. (4) $P_0(x)\propto 1/B(x)\exp\{\int^x[A(x')/B(x')]dx'\}$ satisfy the conditions of the periodicity, the periodic boundary condition, and so on. But if $J=0$, from Eq. (6) we can get $e^{\phi(b)}=1$, i.e., $\phi(b)=0$, and $\phi(b)=0$ is not a general condition of the system studied by us. Thus the zero-current general solution of Eq. (4) is not admissible.

We now turn to a more detailed analysis of Eq. (8). Firstly, let us consider Eq. (1). In general, the random function $\eta(t)$ describes internal thermal noise (additive), and $\xi(t)$ describes external ''nonthermal'' noise (multiplicative). When $g(x)=0$ or $\xi(t)=0$ no transport can occur $(J=0)$. Transport occurring with $g(x)=0$ or $\xi(t)=0$ means that thermal fluctuations are converted into work and implies a violation of the second law of thermodynamics. Equation (8) shows that the condition for the transport of particles is $\phi(b)\neq 0$, i.e.,

$$\int_a^b\frac{A(x)}{B(x)}dx=\int_a^b\frac{f(x)+D_2g'(x)[g(x)+\lambda\sqrt{D_1/D_2}]}{D_1(1-\lambda^2)+D_2[g(x)+\lambda\sqrt{D_1/D_2}]^2}\neq 0. \tag{9}$$

The formula (9) can be further simplified as

$$\int_a^b\frac{f(x)}{D_1(1-\lambda^2)+D_2[g(x)+\lambda\sqrt{D_1/D_2}]^2}\neq 0. \tag{10}$$

In the presence of spatial symmetry, suitably selecting a and b we have $U_0(a+b-x)=U_0(x)$ and $U_1(a+b-x)=U_1(x)$ [i.e., $f(a+b-x)=-f(x)$ and $g(a+b-x)=-g(x)$]. If $\lambda=0$, the left-hand side of Eq. (10) equals zero, which can be easily obtained after using $f(a+b-x)=-f(x)$ and $g(a+b-x)=-g(x)$, now Eq. (10) is false and the current J equals zero; if $\lambda\neq 0$, Eq. (10) is true and the current J does not equal zero. In the case of spatial asymmetry, even if $\lambda=0$, Eq. (10) is true and the current J does not equal zero. Thus the correlation between additive and multiplicative noise and the spatial asymmetry are ingredients for the flux of particles in the spatially periodic system. It has been reported in many references that the spatial asymmetry can induce transport of particles in a spatially periodic system. Here our main point is that the correlation between additive and multiplicative noise can cause net flow even in the absence of spatial asymmetry. In addition, it is clear that when the overdamped Brownian particle is subject to an external constant force, the spatial symmetry will be broken and the particle can exhibit a nonzero net drift speed. If the external force is related to time t, as long as symmetry (spatial symmetry or noise symmetry) is broken the particle will also have a nonzero net speed.

The stationary solution of Eq. (2) is

$$P(x)=N\frac{\exp[\phi(x)]}{B(x)}\oint\exp[-\phi(x')-\phi(b)\theta(x-x')]dx', \tag{11}$$

where $\theta(x-x')$ is the Heaviside step function, and the normalization constant $N=P(a)B(a)/\int_a^b dx\,\exp[-\phi(x)]$. The average velocity

$$\langle\dot{x}\rangle=\langle\langle\dot{x}(x,t)\rangle_f\rangle_x=\langle\langle f(x)+g(x)\xi(t)\rangle_f\rangle_x. \tag{12}$$

According to the Novikov theorem [28], we have (see Appendix)

$$\langle g(x)\xi(t)\rangle_f=D_2[g(x)+\lambda\sqrt{D_1/D_2}]g'(x). \tag{13}$$

From Eqs. (12) and (13) we can obtain the stationary average velocity

$$\langle \dot{x} \rangle_s = \lim_{t\to\infty} \frac{1}{t}\int_0^t \langle\langle \dot{x}(x,\tau)\rangle_f\rangle_x d\tau = \oint A(x)P(x)dx$$
$$= J\{1-\exp[-\phi(b)]\}^{-1}M, \tag{14}$$

where $M=\oint\oint dx dx'[A(x)/B(x)]\exp[\phi(x)-\phi(x') - \phi(b)\theta(x-x')]$. Below we shall investigate the motion of electron pairs in a superconducting junction and the motion of motor proteins along a biopolymer in a the case of environmental perturbation.

III. THE JOSEPHSON JUNCTION

For the superconducting junction, the electric current across the junction is given by $J_c = J_0 \sin\phi$, where ϕ is the phase difference of the superconducting order parameter across the junction and J_0 is a critical electric current. The evolution of the phase difference is described by the equation [29]

$$\frac{\hbar}{2eR}\dot{\phi} + J_0 \sin\phi = I(t), \tag{15}$$

where $I(t)$ is a driving electric current, and R is a resistance. In the case of the environmental perturbation, such as the external vibration, the change of the external temperature, the perturbation of the external electromagnetic field, and so on, the internal structure of the Josephson junction should change. In general, the change is very small. But when the environmental perturbation becomes larger and larger, the change will become clearer and clearer. The change of the internal structure of the Josephson junction should vary the critical electric current. Now we describe it via a stochastic parameter, $J_0 + \sigma\xi_0(t)$, in which $\xi_0(t)$ is a stochastic force, σ is a positive parameter. The driving electric current $I(t) = \eta_0(t)$ is taken as a Gaussian white noise. Then Eq. (15) becomes

$$\frac{\hbar}{2eR}\dot{\phi} + [J_0 + \sigma\xi_0(t)]\sin\phi = \eta_0(t), \tag{16}$$

where we approximately set $\xi_0(t)$ as a Gaussian white noise. The statistical properties of $\xi_0(t)$ and $\eta_0(t)$ are $\langle\xi_0(t)\rangle_f = \langle\eta_0(t)\rangle_f = 0$, $\langle\eta_0(t)\eta_0(t')\rangle_f = 2D_1\delta(t-t')$, and $\langle\xi_0(t)\xi_0(t')\rangle_f = 2D_2\delta(t-t')$. $\eta_0(t)$ is the internal thermal additive noise, and $\xi_0(t)$ the external multiplicative noise. Equation (16) can be simplified as

$$\dot{\phi} = -\omega_0 \sin\phi - \xi(t)\sin\phi + \eta(t), \tag{17}$$

in which $\omega_0 = 2eRJ_0/\hbar$, $\xi(t) = (2e\sigma R/\hbar)\xi_0(t)$, and $\eta(t) = (2eR/\hbar)\eta_0(t)$.

From the above analysis for the origin of the multiplicative noise, we find that here the multiplicative noise represents the external environmental perturbation. In addition, the change of the internal structure of the Josephson junction, which is caused by the environmental perturbation, makes the thermal vibration of the moleculae in the junction vary. The varying (it is a fluctuation) of the thermal vibration of the moleculae should affect the thermal additive noise. Thus here the additive and multiplicative noises are not independent, and there is correlation between them. Here we assume that the correlation function between $\xi_0(t)$ and $\eta_0(t)$ is $\langle\eta_0(t)\xi_0(t)\rangle = 2\lambda\sqrt{D_1 D_2}\delta(t-t')$ $(0 \leqslant \lambda \leqslant 1)$. For convenience, we make Eq. (17) in the dimensionless form and set $\hbar/2e = J_0 = \sigma = R = 1$.

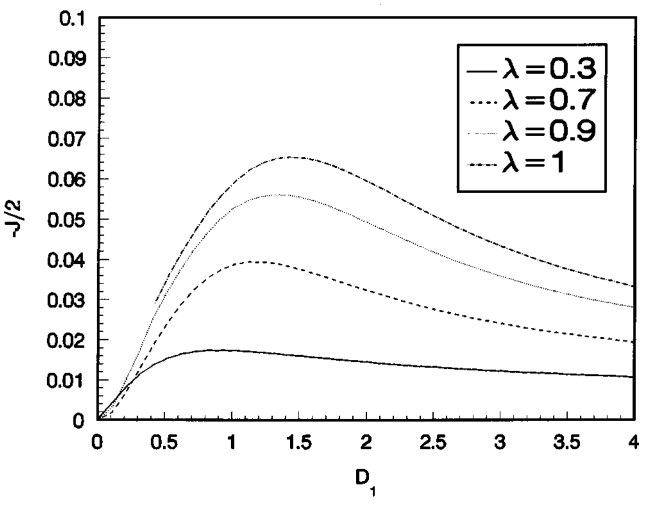

FIG. 1. The negative $J/2$ (J is the probability current) of superconducting junction vs D_1 for different values of λ. $D_2=0.3$, $\omega_0=1$, and $\lambda=0.3$, 0.7, 0.9, and 1. J, D_1, D_2, ω_0, and λ are dimensionless.

In Fig. 1 we plot the negative $J/2$ (J is the probability current) versus the additive noise strength D_1 for different values of λ. Since we have made Eq. (17) in dimensionless form, J, D_1, D_2, ω_0, and λ (in Fig. 1) are dimensionless. Here we set $D_2=0.3$, $\omega_0=1$, and $\lambda=0.3$, 0.7, 0.9, and 1, respectively. When $\lambda=1$, the stationary probability distribution $P(x)$ [see Eq. (11)] will be divergent at the points $x=\sin^{-1}\sqrt{D_1/D_2}+2n\pi$ $(n=0,\pm1,\pm2,\pm3,\ldots)$, so when $D_1<D_2$ we cannot determine J from Eqs. (8) and (11); now we only plot the curve in the case of $D_1>D_2$. Firstly, the figure shows that the probability current is always negative. In addition, the figure also shows (a) in the curve a clear peak value appears; (b) with the increase of λ, the probability current becomes more and more distinct and the peak value moves towards the right and increases.

In Ref. [12], the authors reported that the symmetric noises cannot induce the net voltage in Josephson junctions, namely, the symmetric noises cannot induce the probability current. By the study of the above example we find that if in the system there is a correlation between additive and multiplicative noise, symmetric noise can produce the probability current in Josephson junctions. Now the net voltage is not zero.

IV. THE MOTOR PROTEINS

For the motor proteins, Astumian and Bier proposed a model [6], predictions of which are consistent with the experimental data obtained by Svoboda *et al.* [30] for a single protein molecule moving along a biopolymer.

When a protein molecule with charge moves on a linear highway with a periodic array of fixed charges (along a

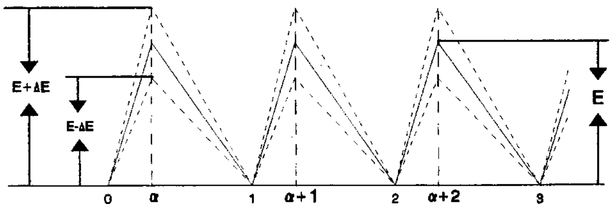

FIG. 2. The fluctuating periodic potential barrier of the motor proteins (in dimensionless form).

biopolymer), its dynamic equation for the position is given by Newton's law as

$$m\ddot{x} = -6\pi\eta a\dot{x} - \frac{\partial}{\partial x}U(x,t), \tag{18}$$

where m is the mass of a molecule, $-6\pi\eta a\dot{x}$ is a viscous drag with η being the viscosity, and a the diameter of the particle, assumed spherical, $U(x,t)$ is a fluctuating potential barrier, which satisfies [6]

$$U(x,t) = U(x) + u(x,t), \tag{19}$$

here the potential at any x fluctuates symmetrically around $U(x)$ and $u(x,t)$ can take the values $+\Delta u(x)$ and $-\Delta u(x)$ [in Fig. 2 we plot $U(x,t)$ in the case of dimensionless form]. We see that the force $F = -\partial U(x,t)/\partial x$ now fluctuates between $F_1^+ = -(E+\Delta E)/\alpha$ and $F_1^- = -(E-\Delta E)/\alpha$ on the interval $(0,\alpha)$, and between $F_2^+ = (E+\Delta E)/(1-\alpha)$ and $F_1^- = (E-\Delta E)/(1-\alpha)$ on the interval $(\alpha,1)$. The flipping rate of the fluctuating potential barrier is γ. Note that we have $\Delta u(0) = \Delta u(1) = 0$ and $U(x,t) = U(x+1,t)$. The fluctuating potential barrier is brought about by the repeated binding of ATP and release of ADP.

If we consider the internal noise (thermal noise), from Eq. (18) we obtain

$$\frac{m\ddot{x}}{6\pi\eta a} = -\dot{x} - \frac{1}{6\pi\eta a}\frac{\partial}{\partial x}U(x,t) + \frac{1}{6\pi\eta a}\eta(t), \tag{20}$$

where $\eta(t)$ represents the Gaussian white noise with zero mean and correlation function $\langle \eta(t)\eta(t')\rangle = 2D_1\delta(t-t')$. When we consider the external environmental perturbation, such as the external vibration, the change of the external temperature, the external electromagnetic field, and so on, the viscosity should change. Now we approximately describe this change via a stochastic parameter, $1/6\pi\eta a + \sigma'\xi(t)$ [$\xi(t)$ is a stochastic force, and σ' is a positive parameter], and only consider the case when the internal and external fluctuations are smaller. As for the case when $\xi(t)$ and $\eta(t)$ are moderate and larger, a detailed theory is under study. Then Eq. (20) becomes

$$\frac{m\ddot{x}}{6\pi\eta a} = -\dot{x} - \frac{1}{6\pi\eta a}\frac{\partial}{\partial x}U(x,t) - \sigma'\xi(t)\frac{\partial}{\partial x}U(x,t) + \frac{1}{6\pi\eta a}\eta(t), \tag{21}$$

in which we neglect the term $\eta(t)\xi(t)$ and retain only the linear noise one. We assume that the multiplicative noise $\xi(t)$ is Gaussian white noise with zero mean and correlation function $\langle \xi(t)\xi(t')\rangle = 2D_2\delta(t-t')$. In addition, the change of the viscosity induced by the environmental perturbation should make the thermal movement of the biopolymers, the proteins, etc. vary, and the varying (it is a fluctuation) of the thermal movement of the moleculae should affect the thermal additive noise. Thus here additive and multiplicative noise are not independent (there is correlation between them). We assume the correlation function between $\eta(t)$ and $\xi(t)$ to be $\langle \xi(t)\eta(t')\rangle = 2\lambda\sqrt{D_1D_2}\delta(t-t')$ $(0 \leqslant \lambda \leqslant 1)$.

For simplicity, we consider the case of dimensionless form. Making the variables $x, t, U(x,t)$, $\xi(t)$, and $\eta(t)$ be dimensionless and setting $1/6\pi\eta a = \sigma' = 1$ and $m = k$ (k is dimensionless), we have (in dimensionless form)

$$k\ddot{x} = -\dot{x} - \frac{\partial}{\partial x}U(x,t) - \xi(t)\frac{\partial}{\partial x}U(x,t) + \eta(t). \tag{22}$$

When $k \ll 1$, one can use the adiabatic approximation $k\ddot{x} \doteq 0$. Now Eq. (22) can be simplified as

$$\dot{x} = -\frac{\partial}{\partial x}U(x,t) - \xi(t)\frac{\partial}{\partial x}U(x,t) + \eta(t). \tag{23}$$

When we neglect the effect of $\xi(t)$, Eq. (23) becomes the model studied in Ref. [6].

For the model proposed by Astumian and Bier, when $\alpha = 1/2$, namely, spatial symmetry, it stops transducing energy. To illustrate that the correlation between additive and multiplicative noise can induce the transport, we only consider the case of spatial symmetry ($\alpha = 1/2$). Going from the Langevin equation (23) to the associated Fokker-Planck equation [26,27,31] for the probability density distribution we find

$$\frac{\partial}{\partial t}\begin{pmatrix} P_i^+(x,t) \\ P_i^-(x,t) \end{pmatrix} = \begin{pmatrix} G_i^+ & \gamma \\ \gamma & G_i^- \end{pmatrix}\begin{pmatrix} P_i^+(x,t) \\ P_i^-(x,t) \end{pmatrix}, \tag{24}$$

where $G_i^+ = -\gamma - F_i^+\partial_x + [D_1(1-\lambda^2) + D_2(F_i^+ + \lambda\sqrt{D_1/D_2})^2]\partial_x^2$ and $G_i^- = -\gamma - F_i^-\partial_x + [D_1(1-\lambda^2) + D_2(F_i^- + \lambda\sqrt{D_1/D_2})^2]\partial_x^2$. $i = 1$ represents the system on the interval (0,1/2) and $i = 2$ represents the system on (1/2,1). The quantities $P^+(x,t)$ and $P^-(x,t)$ are the probabilities at any time t to find the barrier at the $+$ or $-$ configuration, respectively, and the particle at position x.

When $E \gg \Delta E$, we have $F_1 \doteq -2E$ and $F_2 \doteq 2E$. Now we can use the above formulas for Eq. (1). The corresponding Fokker-Planck equation is

$$\partial_t P_i(x,t) \doteq \hat{L}_i P_i(x,t), \tag{25}$$

in which $P_i(x,t) = P_i^+(x,t) + P_i^-(x,t)$, $\hat{L}_i = -F_i\partial_x + B_i\partial_x^2$, and $B_i = D_1(1-\lambda^2) + D_2(F_i + \lambda\sqrt{D_1/D_2})^2$. From Eqs. (8), (11), and (25), it is easy to obtain the probability current

$$J = N[1 - \exp(-\alpha)], \tag{26}$$

where N is the normalization constant for the corresponding probability distribution, and

$$\alpha=-\frac{8E^2\lambda\sqrt{D_1D_2}}{[D_1(1-\lambda^2)+D_2(-2E+\lambda\sqrt{D_1/D_2})^2][D_1(1-\lambda^2)+D_2(2E+\lambda\sqrt{D_1/D_2})^2]}.$$

Equation (26) shows when $E\gg\Delta E$, (a) as long as the correlation between additive and multiplicative noises is not zero, the transport of proteins exists; (b) the probability current has nothing to do with the flipping rate of the fluctuating barrier (now the effect of the flipping rate of the fluctuating barrier on the transport is very small and can be neglected). In Fig. 3 we represent $J/2$ versus λ for $D_1=0.3$, $D_2=0.15$, and $E=0.5$. Since Eq. (23) has been made in the dimensionless form, J, λ, D_1, D_2, and E (in Fig. 3) are dimensionless. From the figure we can find that with an increase of λ the transport of the moleculae becomes more and more distinct.

If $E\sim\Delta E$ or $E\ll\Delta E$, the formulas (8) and (11) are not applicable. But we can use the method in Ref. [6] to calculate the probability current J. In Fig. 4(a) $J/4$ as a function of λ is plotted for $D_1=0.3$, $D_2=0.15$, $E=1$, $\Delta E=0.5$, and $\gamma=3$, 5, 10, and 50, respectively, in dimensionless form. Figure 4(a) shows that when γ increases the flux becomes smaller and smaller. This is because after a flip of the potential surface the probability distribution must take enough time to adjust to the new potential, and the adjustment time is larger than the average flipping time $1/\gamma$. Figure 4(a) also shows that even if $\lambda=0$, there is the flux for the protein moleculae. This is because of the effect of the term $-\xi(t)\times(\partial/\partial x)U(x,t)$ in Eq. (23). When $\lambda=0$, $J/4$ versus D_2, the external noise strength, is plotted in Fig. 4(b) with $D_1=0.3$, $E=1$, $\Delta E=0.5$, and $\gamma=5$, in dimensionless form.

V. CONCLUSION AND DISCUSSION

In conclusion, we have revealed a kind of noise-induced transport of particles. This transport is due to the correlation between additive and multiplicative noise. When the noises in Eq. (1) are O-U (Ornstein-Uhlenbeck) noises, symmetric dichotomous noises, or symmetric Poisson noises, so long as there is a correlation between additive and multiplicative noise, there is the same phenomenon as reported in this paper. Now the correlation between additive and multiplicative noise makes the probabilities of the fluctuations to the right and the left of the potential barrier different, so the transport arises. This correlation breaks the spatial symmetry of the system. The energy in response to the transport originates in a part of the noise's energy, which is determined by the correlation between additive and multiplicative noise. For a motor protein system, we find that in the presence of spatial symmetry, besides the correlation between additive and multiplicative noise, the multiplicative noise is also the origination of the transport. Now the correlation between additive and multiplicative noise and multiplicative noise can both

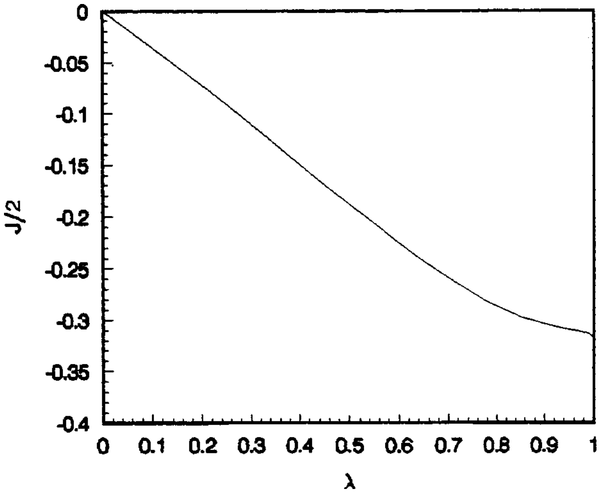

FIG. 3. $J/2$ (J is the probability current) of the motor proteins vs λ in the case of $E\gg\Delta E$. $D_1=0.3$, $D_2=0.15$, and $E=0.5$. J, λ, D_1, D_2, E, and ΔE are dimensionless.

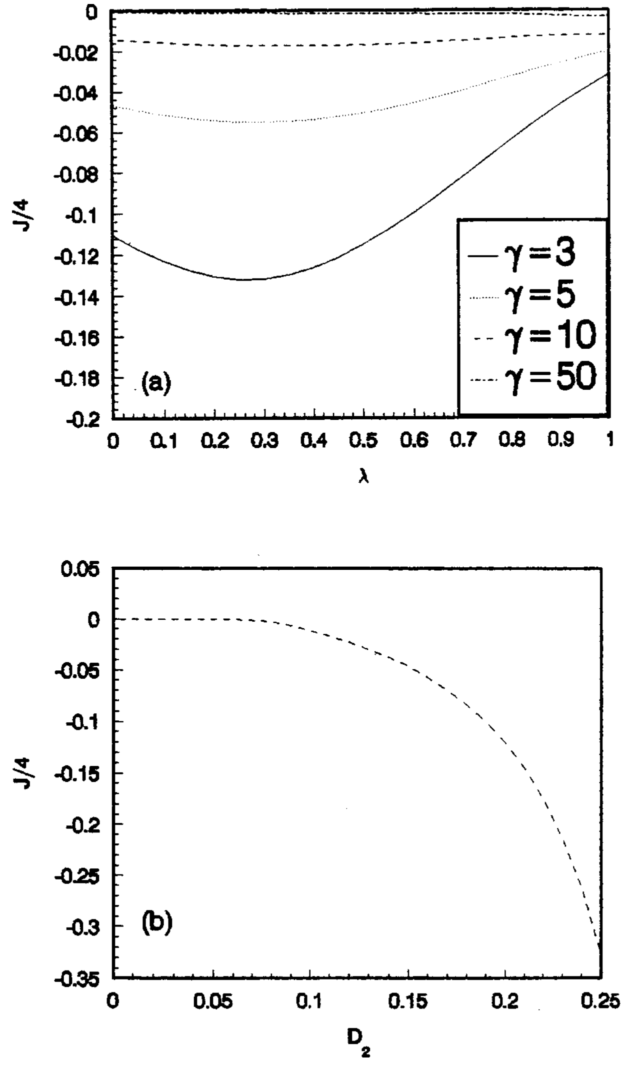

FIG. 4. $J/4$ (J is the probability current) of the motor proteins vs λ for different γ ($\gamma=3$, 5, 10, and 50, respectively) with $D_2=0.15$ (a), and $J/4$ (J is the probability current) of the motor proteins vs D_2 with $\lambda=0$ and $\gamma=5$ (b), in the case of $E\sim\Delta E$. $D_1=0.3$, $E=1$, and $\Delta E=0.5$. J, λ, γ, D_1, D_2, E, and ΔE are dimensionless.

break symmetry (spatial symmetry and noise symmetry).

Finally, it must be stressed that the transport in the superconducting junction and the transport of motor proteins caused by the external environmental perturbation in this paper are only the mathematically and physically theoretical results. And it remains yet to be verified by the experiments whether these transports exist, especially in the case of larger environmental perturbation.

APPENDIX

Equation (1) can be transformed into the following form [26]:

$$\dot{x}=f(x)+[g(x)+\lambda\sqrt{D_1/D_2}]\xi(t)+\eta'(t), \qquad (A1)$$

where $\eta'(t)=\eta(t)-\lambda\sqrt{D_1/D_2}\xi(t)$. The statistical properties of $\eta'(t)$ are $\langle\eta'(t)\rangle=0$, and $\langle\eta'(t)\eta'(t')\rangle=2D_1(1-\lambda^2)\delta(t-t')$. Now the noises $\xi(t)$ and $\eta'(t)$ are no longer correlated.

From Eq. (A1) and the Novikov theorem [28] we can get

$$\langle g(x)\xi(t)\rangle_f=D_2(g(x)+\lambda\sqrt{D_1/D_2})g'(x). \qquad (A2)$$

[1] J. Maddox, Nature (London) **365**, 203 (1993); **368**, 287 (1994); **369**, 181 (1994); S. Leiber, *ibid.* **370**, 412 (1994).
[2] M. O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993).
[3] M. M. Millonas and M. I. Dykman, Phys. Lett. A **185**, 65 (1994).
[4] J. Prost, J.-F. Chauwin, L. Peliti, and A. Ajdari, Phys. Rev. Lett. **72**, 2652 (1994).
[5] C. Doering, W. Horsthemke, and J. Riordan, Phys. Rev. Lett. **72**, 2984 (1994).
[6] R. D. Astumian and M. Bier, Phys. Rev. Lett. **72**, 1766 (1994).
[7] M. Bier, Phys. Lett. A **211**, 12 (1996).
[8] M. Bier and R. D. Astumian, Phys. Rev. Lett. **76**, 4277 (1996).
[9] D. R. Chialvo and M. M. Millonas, Phys. Lett. A **209**, 26 (1995).
[10] A. Ajdari, D. Mukamel, L. Peliti, and J. Prost, J. Phys. (France) **14**, 1551 (1994).
[11] M. C. Mahato and A. M. Jayannavar, Phys. Lett. A **209**, 21 (1995).
[12] M. M. Millonas and D. R. Chialvo, Phys. Rev. E **53**, 2239 (1996).
[13] J. Kula, T. Czernik, and J. Luczka, Phys. Lett. A **214**, 14 (1996).
[14] C. R. Doering, W. Horsthemke, and J. Riordan, Phys. Rev. Lett. **72**, 2984 (1994).
[15] C. Peskin, G. B. Ermentrout, and G. Oster, in *Cell Mechanics and Cellular Engineering*, edited by V. Mow *et al.* (Springer, Berlin, 1994).
[16] J. Rousselet, L. Salome, A. Ajdari, and J. Prost, Nature (London) **370**, 446 (1994).
[17] M. M. Millonas, Phys. Rev. Lett. **74**, 10 (1995).
[18] M. M. Millonas and C. Ray, Phys. Rev. Lett. **75**, 1110 (1995).
[19] M. M. Millonas and D. R. Chialvo, Phys. Rev. Lett. **76**, 550 (1996).
[20] C. R. Doering (unpublished).
[21] J. F. Chauwin, A. Ajdari, and J. Prost, Europhys. Lett. **27**, 421 (1994).
[22] R. Bartussek, P. Hänggi, and J. G. Kissuer, Europhys. Lett. **28**, 459 (1994).
[23] R. Bartussek and P. Hänggi, Phys. Blätter 51 (1995).
[24] J. Luczka, R. Bartussek, and P. Hänggi, Europhys. Lett. **31**, 431 (1995).
[25] M. M. Millonas and D. R. Chialvo, Santa Fe Institute preprint (1996), unpublished.
[26] J.-H. Li and Z.-Q. Huang, Phys. Rev. E **53**, 3315 (1996).
[27] R. Risken, *The Fokker-Planck Equation* (Springer-Verlag, Berlin, 1984).
[28] E. A. Novikov, Zh. Eksp. Teor. Fiz. **47**, 1919 (1964) [Sov. Phys. JETP **20**, 1290 (1965)].
[29] *Physics and Application of the Josephson Effect*, edited by A. Barone (Wiley, New York, 1982).
[30] K. Svoboda, C. F. Schmidt, B. J. Schnapp, and S. M. Block, Nature (London) **365**, 721 (1993).
[31] C. W. Gardiner, *Handbook of Stochastic Method for Physics, Chemistry and the Natural Sciences* (Springer-Verlag, Berlin, 1983).